THE CHINESE UNIVERSITY OF HONG KONG
DEPARTMENT OF MATHEMATICS

MMATH5220 Complex Analysis and Its Applications 2014-2015
Assignment 2

- Due date: 11 Feb , 2015
- Remember to write down your name and student number

1. If $f(z)$ is differentiable at z_0, where $z_0 \neq 0$, show that $f'(z_0)$ can be written as
$$f'(z_0) = e^{-i\theta}(u_r + iv_r)$$
or
$$f'(z_0) = \frac{-i}{z_0}(u_\theta + iv_\theta),$$
where all partial derivatives are evaluated at (r_0, θ_0).

2. Consider the following function
$$f(z) = \begin{cases} (1+i)\dfrac{\mathrm{Im}(z^2)}{|z|^2} & \text{if} \quad z \neq 0, \\ 0 & \text{if} \quad z = 0. \end{cases}$$
   (a) Show that the Cauchy-Riemann equations are satisfied at $z = 0$.
   (b) Is $f(z)$ differentiable at $z = 0$?

3. Find the domains in which the function
$$f(z) = f(x+iy) = |x^2 - y^2| + 2i|xy|,$$
is analytic.

4. Evaluate the integral $\displaystyle\int_\gamma z^2\, dz$, if
   (a) γ is a straight line segment from $z = 2$ to $z = 2i$;
   (b) γ is the major arc of the the circle $|z| = 2$ from $z = 2$ to $z = 2i$.

5. Show that if C is the arc of the circle $|z| = 2$ from $z = 2$ to $z = 2i$ that lies in the first quadrant. By using ML-estimate, show that
$$\left| \int_C \frac{dz}{z^2 - 1} \right| \leq \frac{\pi}{3}.$$

6. If C_R is the arc of the circle $|z| = R$ from $z = R$ to $z = -R$ that lies in the upper half plane. By using ML-estimate, show that
$$\left| \int_{C_R} \frac{z^2}{z^6+1}\, dz \right| \leq \frac{\pi R^3}{R^6 - 1},$$
and hence show that
$$\lim_{R \to +\infty} \int_{C_R} \frac{z^2}{z^6+1}\, dz = 0.$$